

4.8 Using Matrices to Solve Systems of Equations

Matrix Equation

$$\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

2×2 2×1 2×1

$$\begin{cases} x + 3y = 3 \\ x + 2y = 7 \end{cases}$$

Matrix Equation

$$\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

Coefficient Matrix Variable Matrix Constant Matrix

Put the following system into a matrix equation:

$$\begin{aligned} 5x - 3y &= 9 \\ 2x + 3y &= 7 \end{aligned}$$

$$\begin{bmatrix} 5 & -3 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 7 \end{bmatrix}$$

Use Matrix equation to solve

A- coefficient matrix
 X- variable matrix
 B- constant matrix

$$\frac{1}{a} a x = b \frac{1}{a}$$

$$1 x = \frac{1}{a} \cdot b$$

$$x = \frac{1}{a} \cdot b$$

$$A X = B$$

$$A^{-1} \cdot A X = A^{-1} B$$

$$I X = A^{-1} B$$

$$X = A^{-1} B$$

ex:

$$5x + 3y = 13$$

$$4x + 7y = -8$$

$$\begin{bmatrix} 5 & 3 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 4 & 7 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 13 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$$

ex:

$$5x - 2y = 3$$

$$8x + 4y = 3$$

$$\begin{bmatrix} 5 & -2 \\ 8 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} .5 \\ -.25 \end{bmatrix}$$

ex:

$$10x + 5y = 15$$

$$6x + 3y = -6$$

$$\begin{array}{r} 2x + y = 3 \\ -(2x + y = -2) \\ \hline 0 \neq 5 \end{array}$$

$$\begin{bmatrix} 10 & 5 \\ 6 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ -6 \end{bmatrix}$$

\emptyset

Do:

1. $2x + 3y = 10$

$$x + 6y = 32$$

$$\begin{array}{r} -4 \\ 6 \end{array}$$

2. $3x + 4y = -1$

$$6x - 2y = 3$$

$$\begin{array}{r} 1/3 \\ -1/2 \end{array}$$

ex:

$$3x - 2y + z = 0$$

$$2x + 3y - z = 17$$

$$5x - y + 4z = -7$$

$$\begin{bmatrix} 3 & -2 & 1 \\ 2 & 3 & -1 \\ 5 & -1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 17 \\ -7 \end{bmatrix}$$

HW

p205-206

4-6, 24, 26-28